

$0 < t_1 < 2\pi$ .  $K \rightarrow \infty$  corresponds to the case where  $X(t_1)$  is specified.

The Riccati equation for this revised problem is  $\dot{W} = W^2 + \frac{1}{4}$ ,  $W(t_1, t_1) = K$ , so that  $W(\tau, t_1) = -\frac{1}{2} \tan[-\tan^{-1}(2K) + y]$ . Thus  $W(0, t_1)$  is finite if  $0 < t_1 < \pi + 2\tan^{-1}(2K)$ . For  $K = 0$  this gives  $0 < t_1 < \pi$ ; for  $K \rightarrow \infty$ ,  $0 < t_1 < 2\pi$ .

## Comment on "A Combined Visual and Hot-Wire Anemometer Investigation of Boundary-Layer Transition"

ALFONS MICHALKE\*

Deutsche Versuchsanstalt für Luft- und Raumfahrt,  
Berlin, Germany

**K**NAPP and Roache<sup>1</sup> discuss the streakline pattern of the disturbed boundary layer on ogive nose cylinders. With regard to the rolling-up process of the streak-lines in their transition region  $R_1$ , they referred to the paper of Hama<sup>2</sup> who calculated the streakline pattern of a free shear layer perturbed by the neutral disturbance due to the linearized stability theory. Hama found that the streaklines roll up, as if to indicate that the flow develops into vortices. Hama assumed that in the perturbed flow there was no vorticity concentration, and it followed that the rolling-up of streaklines cannot constitute a positive identification of the presence of discrete vortices, and Knapp and Roache<sup>1</sup> argued similarly as well.

This statement of Hama, however, was incorrect, since in the perturbed flow used by him, local concentrations of vorticity existed which essentially corresponded to a one-row vortex street configuration as shown in Refs. 3-5. For amplified disturbances, which were investigated by Knapp and Roache, the rolling-up process of streaklines has been calculated in Refs. 5 and 6. The agreement of the theoretical results with the observed smoke pattern in experiments<sup>7</sup> was good. Furthermore, it was found in Refs. 5 and 6 that the rolling-up process of the streaklines corresponds to a local concentration of vorticity.

I therefore suppose that the streakline pattern observed by Knapp and Roache in their region  $R_1$  will, in fact, indicate a concentration of vorticity, since the streaklines look similar to those observed in free shear layers.<sup>7</sup> Another question, however, is whether these concentrations of vorticity can be characterized as "discrete vortices." I think that this is a question of definition. Surely, these local concentrations of vorticity are not of the type found in a potential vortex or in a Hamel-Oseen vortex. But I should prefer to denote an essential local concentration of vorticity as a discrete vortex, since the effect of a local concentration of vorticity is similar to that of a discrete vortex due to the induction law.

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## Reply by Author to A. Michalke

PATRICK J. ROACHE\*

Sandia Laboratories, Albuquerque, N. Mex.

**W**E genuinely appreciate Michalke's Comment<sup>1</sup> and this opportunity to discuss a point that has clouded the literature on boundary-layer transition for years. A long-standing semantic confusion exists between Michalke<sup>1,2</sup> and Hama<sup>3,4</sup> over the imprecise use of the words "vorticity maximum" and "vorticity concentration." Both use the word "concentration" with what they want to characterize as a discrete vortex. But Michalke<sup>2,5</sup> has used the words "maximum," "extremum," "peak," and "concentration" interchangeably, whereas Hama<sup>3,4</sup> clearly has used "maximum" and "concentration" with different meanings. Hama<sup>3</sup> certainly did not "pretend"<sup>1</sup> that there was no vorticity concentration; he plotted out both the mean and the fluctuating vorticity distributions. Also, he says "... the flow ... does not explicitly exhibit singularities. The mean vorticity ... as well as the fluctuating vorticity ... are both broadly distributed in  $y$  direction ... and the latter fluctuates sinusoidally; there is [sic] no higher harmonics in the fluctuations or no vorticity concentrations in this flow." Hama uses "concentration" and "discrete vortex" only in the sense of a singularity, at least in Ref. 3. But Michalke<sup>5</sup> quite successfully interprets free shear layer instability in terms of vortex induction, and so refers to the smooth vorticity extrema as "concentrations."

This leads to the problem of definition of a "discrete vortex," the problem being that no singularity can exist in viscous flow; and by Hama's criteria in Ref. 3, no discrete vortex can exist. This is clearly an inadequate definition, as recognized by all parties, including Hama.<sup>4</sup> Thus exists the present argument (also treated by Hama and Nutant,<sup>4</sup> Klebanoff, Tidstrom, and Sargent,<sup>6</sup> and Kovasznay, Komoda, and Vasudeva<sup>7</sup>) over whether the discrete vortex is formed before or after the three-dimensional deformation of the  $R_1$  wave front.

Consider other definitions. One textbook<sup>8</sup> simply states that "a region containing vorticity is called a vortex." (Thus, the stable laminar boundary layer itself is a vortex.) But it seems that everyone involved envisions "vortex" to require some closed streamlines and some sort of vorticity maximum. Now the so-called Tollmien-Schlichting waves exhibit these characteristics (see Fig. 16.14 of Schlichting<sup>9</sup>), but in customary usage they have not been called vortices. Somehow, the vorticity has not been concentrated enough to warrant the characterization as a discrete vortex. And although vague agreement has been reached among different investigators on the distinction, the fact is that no one has ever quantified his criterion.

There has been another criterion widely used by experimentalists<sup>6,7</sup> to distinguish wavy disturbances of the Toll-

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\* Technical Staff Member, Advanced Weapons Aerodynamics Division, Aerothermodynamics Department. Member AIAA.

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\*Senior Scientist, Institut für Turbulenzforschung.